Brexit as a Bargaining Game: Static and Dynamic Models of UK–EU Trade Negotiations

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I. Introduction

On June 23, 2016, 51.9% of voters in the United Kingdom voted in favor of the United Kingdom's departure from the European Union, known as the Brexit referendum.[1] Brexit, which did not officially go into effect until January 31, 2020, marked a pivotal moment in Europe's history that led both entities down a long road of tense, extensive negotiations. To understand the consequences of Brexit, it is important to comprehend the dynamics of the two entities before the vote. The United Kingdom joined the European Union in 1973 and from it enjoyed access to the European Union's single market, which allowed for the free movement of capital, people, goods, and services across internal borders. The economies of the United Kingdom and the European Union benefited greatly from this agreement because this unification opened opportunities for trade, investment, and economic cooperation without barriers such as tariffs or quotas. However, this membership also came with some requirements that proponents of Brexit saw as damaging to the United Kingdom. This included contributing a proportional amount to the European Union budget, adhering

[1] "EU Referendum Results," BBC News, accessed May 10, 2024, `https://www.bbc.co.uk/news/politics/eu_referendum/results`.

to European Union regulations, and respecting the authority of the European Court of Justice. The United Kingdom, given the size of its economy and progressive advancements, was an integral part of the European Union's economy and budget plan. Below, Figure 1 shows the net contribution/receipts of each country of the European Union for the 2017 budget as well as the amount per capita.

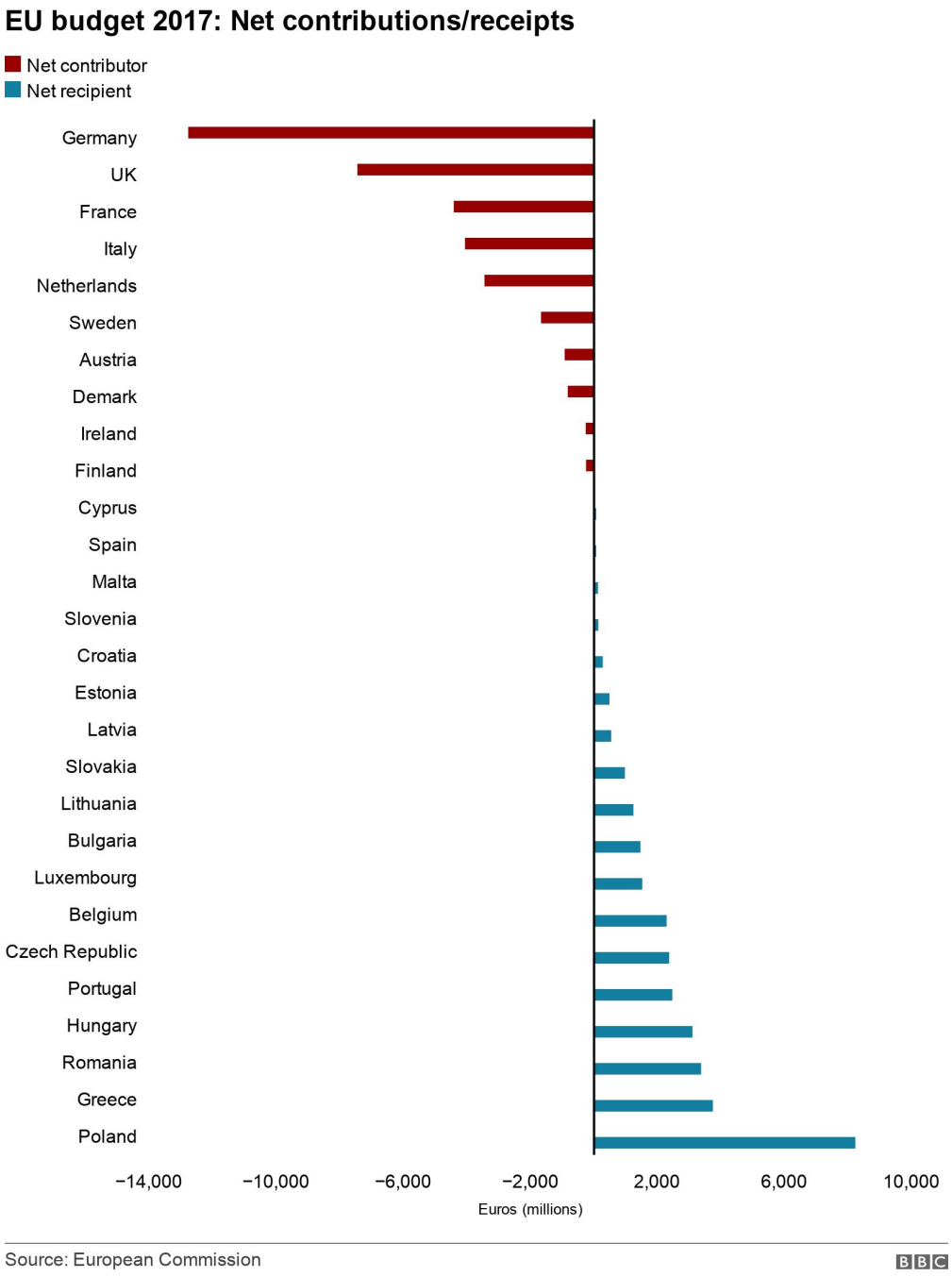

Figure 1: EU budget 2017 — net contributions/receipts (left) and contributions/receipts per capita (right). Source: European Commission; Eurostat, via BBC.

From Figure 1, we see that the United Kingdom was the second largest net contributor to the European Union at €7.43bn (£6.55bn).[2] Although each country paid the same

[2]Tamara Kovacevic, "EU Budget: Who Pays Most in and Who Gets Most Back?," BBC News, May 27, 2019, `https://www.bbc.com/news/uk-politics-48256318`.

proportion of its national income to the budget, the amount each received was dependent on their necessity according to a range of European Union projects and initiatives. For example, a large portion of the budget every year went to agriculture and the development of poorer areas of the European Union in support of the Cohesion Policy. This inevitably resulted in poorer countries and those with many farms receiving larger portions of the budget. Although the order of contribution changes when comparing the contribution per capita, the United Kingdom remained one of the top five contributing countries, with net contributions of €112.85 per capita. Evidently, the United Kingdom was a large part of the European Union's economy. Supporters of Brexit felt that the United Kingdom was under-appreciated, disrespected, and not gaining enough in return for the role they played in the Union. Proponents also argued that Brexit would help the United Kingdom attain greater sovereignty within Europe, control immigration, and increase the ability to independently negotiate trade agreements with other countries. They wished to set their own regulations, control their borders, and create trade partnerships without being restricted by the European Union. However, opponents of Brexit emphasized the benefits of remaining within the European Union, such as access to a large market, freedom of movement, collaborative research and development programs, and shared security initiatives.

Once British citizens had voted on the Brexit referendum, negotiations between the European Union and the United Kingdom were marked by contrasting priorities and complex dynamics. Key focuses during the negotiations included trade deals, fisheries, the Northern Ireland border, governance mechanisms, and the role of the European Court of Justice. The United Kingdom's officials sought a comprehensive trade deal that maintained free market access while allowing freedom in global trade deals, regulatory autonomy, and border control. In contrast, the European Union stressed the integrity of its single market, highlighting the fact that complete freedom for the United Kingdom in terms of trade would dismantle the very foundation of the European Union market. In addition, the European Union emphasized the importance of regulatory alignment and the need to strive for level playing fields in

aspects such as state aid, competition, and environmental standards. All in all, the Brexit situation allows for an interesting examination from a game theory perspective given the stubborn leadership on both sides of the negotiation table. In our paper, we explore how Brexit, specifically the debates surrounding a trade agreement, can be modeled as different games to better understand the various dynamics and how the role of negotiation tactics affected varying outcomes.

II. Standard Game

In the standard game, we have the following setup:

Formal Game Representation:

$$\begin{aligned} \text{Players:} \quad & \{UK,\ EU\} \\ \text{Actions:} \quad & \{\text{Hawk},\ \text{Dove}\} \\ \text{Payoffs:} \quad & \{\, u(H,D),\ u(H,D)\,\} \end{aligned}$$

We then have the following payoff matrix:

		EU	
		Hawk	**Dove**
UK	**Hawk**	$x - C_H^U,\quad y - C_H^E$	$P_1,\quad -C_D^E$
	Dove	$-C_D^U,\quad P_2$	$e,\quad t$

Table 1: Standard (generalized) payoff matrix. UK payoffs listed first in each cell.

In this matrix, our players, the United Kingdom and the European Union, have the choice to either play Hawk or Dove, representing respectively whether they chose to leave the agreement between the two entities, or stay in the agreement. In this generalized form,

C_H^U and C_H^E represent the costs incurred by the United Kingdom and by the European Union as a result of being Hawkish. We can think of these costs as trade lost for each entity from the loss of a free trade agreement or as the taxes that now will be required in order to conduct trade without the agreement. C_D^U and C_D^E represent the costs incurred by the United Kingdom and the European Union when their opponent plays Hawk but they play Dove. It is important to note that the cost of both the United Kingdom and the European Union face larger costs under Dove than under Hawk, that is, $|C_D^i| > |C_H^i| \quad \forall\, i \in \{U, E\}$. We can think of these costs as loss of trade or as a measure of the reputation loss by the countries that showed weakness by playing a softer move than their opponent. Next, P_1 and P_2 represent nationalistic pride utils that the United Kingdom and European Union receive if they play Hawk when their opponent plays Dove. We can think about this as a reputation gain in geopolitical negotiations in the future as the entities will appear stronger in the future and therefore have reputational leverage. Similarly, x and y are also nationalistic pride utils that the United Kingdom and the European Union receive when they both play Hawk. We can interpret these utils as being the geopolitical benefit and reputational gain that the entities receive from not budging when their opponent is threatening them. Finally, e and t represent the benefit endowed on the United Kingdom and the European Union when they both play Dove. We can interpret these as the trade benefits that both entities receive when they can both enjoy free trade between one another. This util can also be interpreted as a collaboration util.

The mixed strategy equilibrium for this game when the United Kingdom plays Hawk with probability r and Dove with probability $(1 - r)$ and the European Union plays Hawk with probability q and Dove with probability $(1 - q)$ is as follows:

$$q^* = \frac{e - P_1}{e - P_1 + x + \Delta C^U}, \tag{1}$$

$$r^* = \frac{t - P_2}{t - P_2 + y + \Delta C^E}. \tag{2}$$

The full mixed strategy equilibrium can be written as

$$\text{MSE} = \left\{ \left(H,\ \frac{t-P_2}{t-P_2+y+\Delta C^E}\ ;\ D,\ 1-\frac{t-P_2}{t-P_2+y+\Delta C^E} \right);\ \left(H,\ \frac{e-P_1}{e-P_1+x+\Delta C^U}\ ;\ D,\ 1-\frac{e-P_1}{e-P_1+x+\Delta C^U} \right) \right\}. \tag{3}$$

In the numerator of our r^* and q^*, $(t-P_2)$ and $(e-P_1)$ represent the losses that the United Kingdom and the European Union face for being Hawkish given their respective rival plays Dove. The denominator also has $(t-P_2)$ and $(e-P_1)$, the losses that the entities face for being Hawkish, added to x plus the differential costs between the trade lost by being Hawkish and being Doveish. The differential costs in both r^* and q^* are positive because $|C_D^i| > |C_H^i| \ \ \forall\, i \in \{U, E\}$, which implies that one of the two following scenarios takes place: either e is strictly or weakly greater than P_1, or e is less than P_1. In the second case, this would require x to be greater than the difference between e and P_1, in order to keep q^* positive. Similar logic follows for r^*.

Taking the partial derivative of these probabilities with respect to our pride utils (P_1 and P_2), we have the following expression:

$$\frac{\partial q^*}{\partial P_1} = \frac{(-1)k + (e-P_1)}{k^2} = \frac{-(x+\Delta C^U)}{k^2} < 0, \tag{4}$$

where k is the denominator of our mixed strategy equilibrium probabilities. Because this game is symmetric, a similar derivative for r^* follows. This expression shows us that as the level of an opponent's nationalistic/organizational pride increases, the probability of one playing Hawk decreases.

When r^* and q^* are both equal to $\frac{1}{2}$, the scenario in which there is maximized uncertainty,

we have the following expression:

$$r^* = q^* = \tfrac{1}{2}, \tag{5}$$

$$r^* = \frac{e - P_1}{e - P_1 + x + \Delta C^U} = \tfrac{1}{2}, \tag{6}$$

$$e - P_1 = x + \Delta C^U. \tag{7}$$

$$\underbrace{e - P_1}_{\text{Loss from being Hawkish}} = \underbrace{x}_{\text{Reputation Gain}} + \underbrace{\Delta C^U}_{\text{Cost Cutting by being Hawkish}}$$

That is, when the probability of playing Hawk and Dove are both $\frac{1}{2}$, the losses incurred from being Hawkish are equal to the reputational gain plus the cost cutting from being Hawkish.

III. Sequential Game

Next, we will change the dynamics of the game to better represent what actually took place during the Brexit negotiations and to examine how mutating aspects of the game lead to differing results. We assign simple numbers to some of the variables in our matrix to allow for a game that can be better interpreted as a real-life situation. First, note that the absolute value of C_D^i is greater than the absolute value of C_H^i where i represents either the United Kingdom (U) or the European Union (E), mathematically denoted as $|C_D^i| > |C_H^i| \quad \forall\, i \in \{U, E\}$. Given i's rival plays Hawk, i incurs a strictly higher cost from playing Dove than playing Hawk. Playing Dove implies the cost of complete dominion of i by its rival. Playing Hawk on the contrary initiates collateral damage from trade wars. That is, if the United Kingdom plays Dove, their loss of trade cost is higher than if they played Hawk. Moving to the values of the variables, C_H^U and C_H^E are each assigned to be 1, C_D^E and C_D^U to be 2, and both e and t to also be 2. Thus, the payoff matrix follows:

Sequential Game: $|C_D^i| > |C_H^i| \; \forall \, i$

		EU: Hawk	**EU: Dove**
UK	**Hawk**	$x-1, \quad y-1$	$P_1, \quad -2$
	Dove	$-2, \quad P_2$	$2, \quad 2$

Table 2: Sequential game payoff matrix with assigned cost values. UK payoffs listed first in each cell.

This is a sequential game, meaning that one player moves first and then the next player chooses their action knowing the other player's move. In this case, we assign the United Kingdom to move first, followed by the European Union. The following tree illustrates the action choices of each entity in order of their move position.

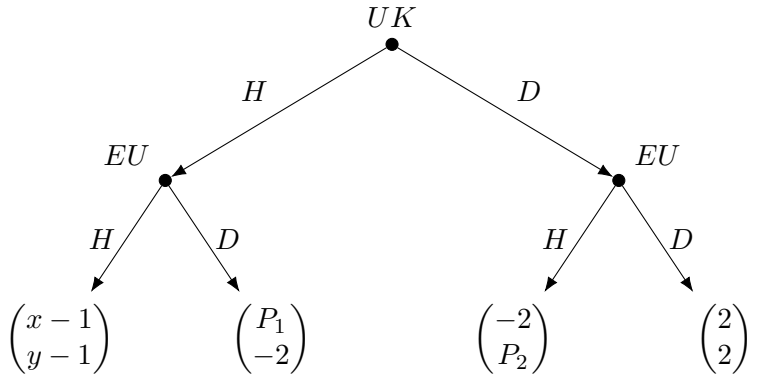

Figure 2: Extensive-form (game tree) representation. The UK moves first; the EU responds. Payoff vectors list the UK payoff on top and the EU payoff on bottom.

In this newly illustrated game, there are still the same two players $\{UK, EU\}$, the histories are $\{\Phi, H, D\}$, the terminal histories are $\{HH, HD, DH, DD\}$, and the strategies for the United Kingdom and the European Union are $\{H, D\}$ and $\{HH, HD, DH, DD\}$ respectively. This tree also has 3 sub-games, represented by the bottom left European Union game, the bottom right European Union game, and the entire tree. From this we can construct the

new matrix that takes into account the sequential moves.

		EU			
		HH	**HD**	**DH**	**DD**
UK	**H**	$\underline{x-1},\ \underline{y-1}$	$\underline{x-1},\ \underline{y-1}$	$P_1,\ \underline{-2}$	$\underline{P_1},\ \underline{-2}$
	D	$-2,\ \underline{P_2}$	$\underline{2},\ \underline{2}$	$-2,\ \underline{P_2}$	$\underline{2},\ \underline{2}$

Table 3: Normal-form representation of the sequential game. Solid underlines denote strict best responses; dashed underlines denote best responses contingent on the magnitude of the relevant variables.

The single line underline represents strict dominance of that action over the other, for example, holding the European Union's action of HH constant, the United Kingdom's payoff from H is greater than D ($x - 1 > -2$) because the geopolitical benefit and reputational gain from playing Hawk (x) is greater than 0. The dotted lines under the payoffs indicate that the choice of one player's action holding the other's constant is dependent based on the magnitude of certain variables. For example, given the European Union's DD, if p_1 is greater than 2, then the United Kingdom will choose Hawk. However, if it is less than 2, then the United Kingdom will choose Dove. For (H, HH) to be an equilibrium, $x - 1 > -2$ (or $x > -1$) and $y - 1 > -2$ (or $y > -1$), and for (H, HD) to be an equilibrium, $x > 3$ and $y > -1$. Similarly for (D, HD) to be an equilibrium, $2 > x - 1$ (or $x < 3$) and $p_2 < 2$, and for (D, DD) to be an equilibrium, $p_1 < 2$ and $p_2 < 2$ which means the prestige of the UK and EU must be below a threshold of 2. This is intuitive as prestige leads to aggression.

Thus, we arrive at the following Nash Equilibria:

$$\text{Nash Equilibria:} \quad \{\,(H, HH);\ (H, HD);\ (D, DD);\ (D, HD)\,\}. \tag{8}$$

The Subgame Perfect Nash Equilibrium follows by using backward induction on the tree

game. Starting with the two European Union subgames, the European Union will choose H in the left subgame because their payoff of $(y-1)$ is greater than (-2) since $y > 0$. However, in the right subgame, since we do not know the comparison between p_2 and 2, we will proceed with both as options in separate games. This arrives at the following two trees where it is now the United Kingdom's choice based on their knowledge of how the European Union will react:

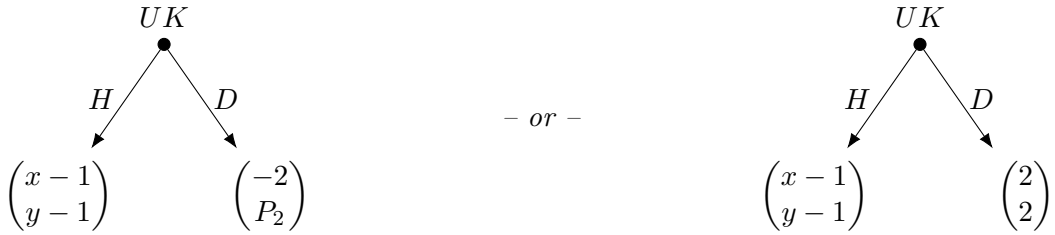

Figure 3: Reduced trees after backward induction over the EU subgames. Left: EU committed to HH. Right: EU committed to HD.

In the left tree, which assumes the European Union chose HH, the United Kingdom will choose H because $(x-1)$ is greater than (-2), since we are assuming $x > 0$. This results in the SPNE of (H, HH). In the right tree, which assumes the European Union chose HD, the United Kingdom doesn't have a dominant choice since the magnitude of x is unknown. Two possible SPNEs result: (H, HD) or (D, HD). In the case of EU playing HD, if x is greater than 3, then the United Kingdom will choose H, resulting in (H, HD). But if $0 < x < 3$, then the SPNE will be (D, HD). Generally, if e and t do not equal 2, then the United Kingdom will choose H when $x > 1 + e$ and will choose D when $x < 1 + e$.

Connecting these SPNE with the Nash Equilibria, we have three SPNE: (H, HH), (H, HD), and (D, HD), which are also Nash Equilibria. (D, DD) is the only Nash Equilibrium that cannot be an SPNE because DD is not a credible threat for the European Union—meaning that the European Union would never play DD no matter what the United Kingdom plays. If the United Kingdom plays H, then the European Union would not choose D because -2 is always less than $(y-1)$. Therefore, the solution to this sequential game is (H, HH), (H, HD), and (D, HD), resulting in the following implications and intuition of possible real-

world outcomes. The solution (H, HH) represents an "all out war" between the two entities, meaning that both sides of the negotiation table are playing tough and are refusing to back down. This has many consequences as these countries limit their trade abilities and access to each other's market. Further, it sets a poor precedent for any future negotiation and good will. On the other hand, both (H, HD) and (D, HD) represent "quid pro quo" where both entities can come to an agreement if one side negotiates harshly then the other will also, and if one does not then the other will not either.

IV. Implications and Conclusions

In the real-life scenario, the United Kingdom pulled out of the European Union, a situation which would be reflected by both entities playing Hawk in our game. However, the stakes were not symmetric as our model would suggest. The United Kingdom lost free trade with the other members of the European Union, a choice that their economy suffered from greatly. The other members of the European Union lost free trade with the United Kingdom, which also caused a loss in trade, though a significantly smaller loss in trade than the United Kingdom's. Introducing trade tariffs and customs checkpoints within the United Kingdom increased frictions to trading significantly, and "An official forecast by the government's own independent analysts estimates that the new UK-EU trading relationship will 'reduce long-run productivity by 4% relative to remaining in the EU.'"[3] In our game, this is represented by the outcome $\{\text{Hawk}, \text{Hawk}\}$ in our static game and in our dynamic game, the entities chose to play $\{H, HH\}$, the all-out war scenario, rather than $\{H, HD\}$ or $\{D, HD\}$, the quid pro quo scenarios. The exchange rates for the British pound to plummeted after Brexit, making the pound the weakest it had ever been. The British pound's strength has still not recovered to its pre-Brexit strength. Interest rates within the United Kingdom have also been substantially raised, which has deeply hurt the country's macroeconomy.

Intuitively, once the EU saw the referendum, they knew the UK was more likely to play

[3] "Post-Brexit Guide: What's been the impact — and how did it happen?" Euronews, December 29, 2022.

Hawk. In the sequential game this matters directly: the UK chooses Hawk precisely when its pride exceeds the threshold $P_1 > 2$, so a more prideful, more Hawkish UK pushes the equilibrium toward the all-out-war outcome $\{H, HH\}$ rather than a quid pro quo. The EU, anticipating a Hawkish UK and unwilling to concede free-trade access, responds in kind. We also can explore the leaders behind Brexit on the United Kingdom's side, particularly Boris Johnson, as being an embodiment of a UK whose pride sat above the prescribed threshold, and who pushed for Brexit after seeing that their opponent, the European Union, would not allow them to maintain free trade were they to leave the European Union. In all, nationalistic pride creates collateral damage in the form of trade loss.

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